Superpositions of Squeezed States and their Interaction with Two-Level Atoms

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Received September 28, 1994

We present a study of some aspects of the properties of single mode superpositions of squeezed states as well as their interaction with a single two-level atom, in the framework of the Jaynes-Cummings model. We compare the behavior of systems having two particular types of superpositions as initial fields, corresponding to different orientations of the constituent states in phase-space. We investigate the collapses and revivals of the atomic inversion, as well as the field purity and its connection with the evolution of the Q-function in phase-space.

I. Introduction

The study of optical macroscopic superposition states (or "Schrödinger cats") has been rather fruitful over the past few years[1−5]. The natural candidates for such states are linear superpositions of coherent states, which exhibit quite unusual statistical properties, such as quadrature squeezing, strong oscillations in their photon number distribution, as well as sub-Poissonian character[1,2]. At the same time they are relevant to Schrödinger's cat problem[6]., especially because the coherent states are the "most classical" pure states allowed in quantum mechanics. More recently, however, a new class of superposition states has been introduced: the superpositions of squeezed coherent states[7−9]. Because of the already intrinsic nonclassical properties of the constituent states (squeezed states), the resulting superpositions should exhibit even sharper nonclassical features than the superposition states using coherent states. Nevertheless, despite the possibility of constructing squeezed coherent states with a "macroscopic number of photons", their superpositions still can not be considered legitimate Schrödinger cats, just because each component state is nonclassical, unlike the superposition of coherent states, where each component state is "quasi-classical".

The main purpose of this paper is to investigate some of the fundamental aspects of the interaction of superpositions of squeezed coherent states with two-level atoms in the framework of the exactly-solvable one-photon Jaynes-Cummings model (JCM)[10]. The model, a single two-level atom interacting with a single mode of the electromagnetic field in an optical cavity, allows a deep inspection of the process of interaction of nonclassical light with matter, and studies involving squeezed states[11,12,9], as well as superpositions of coherent states[5] as initial fields, for instance, can be found in the literature. Because the superpositions considered here present both a coherent amplitude and squeezing in each constituent state, as we are going to see, there will be an interesting blend of features

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due to each fact, bringing important consequences on the dynamics of both the atom and the field. Due to the peculiar phase-sensitive noise properties[7] of the squeezed states of light, we are able to construct an infinite number of superposition states given a coherent amplitude. In this paper we will be concerned only with two of these possible combinations, i.e., superpositions of squeezed states with both constituent states having squeezing parameters (equal and real) either positive or negative, corresponding, to super-Poissonian and sub-Poissonian states, respectively. The coherent amplitude will also be chosen real, i.e., the states will lie on the real (x) axis in phase space. We have chosen these two cases not only because they represent extreme situations, but also because some methods of production as well as properties of such fields have been already discussed in the literature[8,9]. We will show that the evolution of the atom (field) will be substantially different in each case, and that the destruction of the nonclassical properties due to the interaction will be less effective if the squeezing parameter of the constituent states of the initial field is positive, and what corresponds, in the examples studied here, to sub-Poissonian photon statistics. This also means that the component states are squeezed in the $\hat{X}$ quadrature, while lying on the x axis.

This work is constituted by the following main parts: in Section II we will review a few properties of superpositions of squeezed coherent states. In Section III and IV we will discuss the dynamics of the atom and field, respectively, and in Section V we will summarize our conclusions.

II. Some properties of superpositions of squeezed coherent states of light

II.1 Squeezing and the Wigner function

The states we consider here as input fields, i.e., quantum superpositions of squeezed coherent states of light, are defined as:

$$|\Psi\rangle = \mathcal{N}^{1/2}\left(|\alpha, r\rangle + |-\alpha, r\rangle\right), \qquad (1)$$

where $|\alpha\rangle = \hat{D}(\alpha)\hat{S}(r)|0\rangle$, with $\hat{D}(\alpha) = \exp(\alpha\hat{a}^\dagger - \alpha^*\hat{a})$ and $\hat{S}(r) = \exp[r/2(\hat{a}^2 - \hat{a}^{\dagger 2})]$ being the displacement and squeezing operators, respectively[7]. The coherent amplitude α will be chosen real and positive, and the squeezing parameter r, will be either positive or negative (real), depending on the orientation wanted for the constituent states $|\alpha, r\rangle$ and $|-\alpha, r\rangle$. The constituent states are squeezed with respect to the quadrature operators,

$$\hat{X} = \frac{\hat{a} + \hat{a}^\dagger}{2}; \qquad \hat{Y} = \frac{\hat{a} - \hat{a}^\dagger}{2i}; \qquad [\hat{X}, \hat{Y}] = i/2, \qquad (2)$$

which means that one of them displays fluctuations below the vacuum level, for instance, $\langle \Delta\hat{X}^2\rangle < 1/4$ for $r > 0$ and $\langle \Delta\hat{Y}^2\rangle < 1/4$ for $r < 0$. Nevertheless, because of the commutation relation in equation (2), the uncertainty relation is always preserved, i.e., $\langle \Delta\hat{X}^2\rangle\langle \Delta\hat{Y}^2\rangle \geq 1/16$.

It is interesting to note that superpositions of squeezed states exhibit a higher degree of squeezing than the constituent states[8], and this is due to the quantum interference arising from the superposition process. In order to better illustrate the squeezing properties of the superposition state in equation (1), it is interesting to calculate its Wigner function. The Wigner function is one of the possible quasiprobability distributions in phase space that can be defined within quantum mechanics, and its usefulness in the description of (nonclassical) quantum optical fields has been widely recognized, particularly concerning quantum superposition states[2]. If a field is prepared in a quantum state described by $\hat{\rho}$, we can define an infinite number of (s-parametrized) quasiprobability distributions in phase space as:

$$F(\beta; s) = \frac{1}{\pi^2}\int d^2\xi\, C(\xi; s)\, \exp(\beta\xi^* - \beta^*\xi), \qquad (3)$$

where the quantum characteristic function is:

$$C(\xi; s) = \mathrm{Tr}[\hat{D}(\xi)\hat{\rho}]\exp(s|\xi|^2/2). \qquad (4)$$

Here $\beta = x + iy$, with (x, y) being the c-numbers corresponding to the quadratures $(\hat{X}, \hat{Y})$, respectively. For particular values of s we obtain the well known distributions, e.g., for $s = 0$ we have the Wigner function, and for $s = -1$ the (Husimi) Q-function. The Wigner function of our superposition state (1), is given by:

$$W(x,y) = \frac{\mathcal{N}}{\pi}\exp(-2y^2e^{-2r})\Big\{\exp[-2(x-\alpha)^2e^{2r}]+\exp[-2(x+\alpha)^2e^{2r}] \\ +2\cos(4\alpha y)\exp(-2x^2e^{2r})\Big\}$$

where $\mathcal{N}$ is a normalization factor close to 1/2.

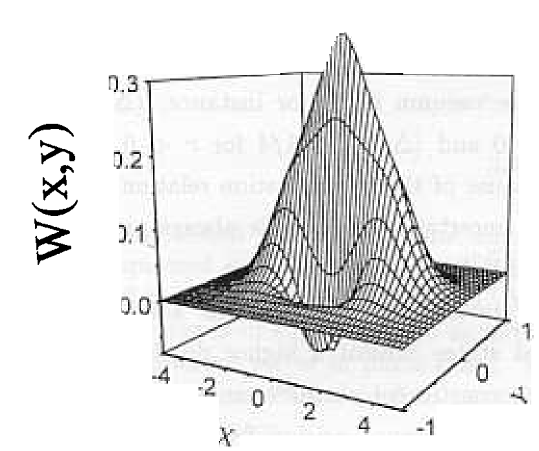

Figure 1: Wigner function of a superposition of two squeezed coherent states with $r = -0.7$ and $\alpha = 2$.

We can already see in expression (5), that for $r < 0$, for instance, there is squeezing in the variance of $\hat{Y}$ for each constituent state. The resulting state also presents squeezing. This is better illustrated in Fig. 1, where we plot the Wigner function [in equation (5)] taking $r = -0.7$ and $\alpha = 2$. We note an elongated structure, which is basically formed by the two peaks corresponding to the individual squeezed states plus the interference structure, the latter being responsible for the nonclassical effects as well as for the negativity of the Wigner function.

Another quantity to characterize the field statistics that often shows nonclassical effects is the photon number distribution. The atomic response in the JCM is extremely sensitive to the photon number distribution of the initial field, and therefore it would be worth to examine it in some detail.

II.2 Photon number distribution

The photon number distribution is defined as:

$$P_n = \langle n|\hat{\rho}|n\rangle,$$

i.e., as the probability of having n photons in a quantum state described by $\hat{\rho}$. In our case, for the state in equation (1), the photon number distribution can be written as:

$$P_n = \frac{\mathcal{N}}{n!\cosh r}\left(-\frac{1}{2}\tanh r\right)^n \exp[-\alpha^2(1+R\tanh r)] \\ \times\Big[\mathcal{H}_n(A)^2+\mathcal{H}_n(-A)^2+2\mathcal{H}_n(A)\mathcal{H}_n(-A)\Big],$$

where $R = r/|r|$ (equals -1 or 1 in our case), and $\mathcal{H}_n(A)$ is a Hermite polynomial with argument

$$A = \frac{\alpha\,(1+R\tanh r)}{(2R\tanh r)^{1/2}}. \qquad (8)$$

Because of the complexity of the expression above, it is suitable to depict P_n for both r positive and negative. In Fig. 2a we have a plot of P_n with $\alpha = 5$ and $r = 1$. We can immediately identify two main features: "microscopic" oscillations (or non-existence of odd photon numbers) and "macroscopic" oscillations for $n > \alpha^2$. The former ones are due to interference in the "coherent part" of the superposition, i.e., just because one of the constituent states has coherent amplitude α, while the other has amplitude $-\alpha$. This is a well-known property[1,2] of such superposition states. The latter oscillations are also a well-known feature of the photon number distribution of squeezed states with $r > 0$. We see that, in superpositions of squeezed coherent states both effects are present, as we would expect. In Fig. 2b, we have a plot of P_n for $\alpha = 5$ and $r = -1$. In this case, the constituent states are oriented in such a way that squeezing occurs in the $\hat{Y}$ quadrature, and the distribution is broader than in the former case (super-Poissonian). The "macroscopic" oscillations are not present, but the "microscopic" ones are, i.e., there is absence of odd photon numbers. This absence is due again to the fact that the coherent amplitudes in the constituent states have the same absolute value but oposite signs[1,2].

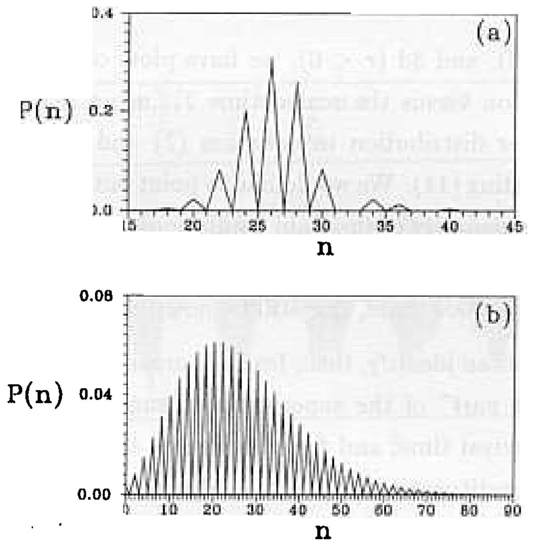

Figure 2: Photon number distribution of a superposition of two squeezed coherent states with a) $r = 1$ and b) $r = -1$. In both cases $\alpha = 5$.

As we are going to show in the next Sections, these peculiarities of the initial field will have a strong influence on both the atomic and field dynamics.

III. Atomic response to the field

III.1 Approach to the problem

Here we are going to apply the method already used to treat similar problems[14,5,9]. We are interested in the evolution of the purity of the quantum states involved, and this favors, because of its generality, a solution in terms of the density operator. The Jaynes-Cummings model[10] consists of a two-level atom with ground state $|g\rangle$ and an excited state $|e\rangle$ placed inside a lossless cavity and interacting with a quantized single mode of the electromagnetic field. Its Hamiltonian in the rotating wave approximation, where only energy conserving terms are kept, may be written as:

$$H = \frac{1}{2}\hbar\omega_0\sigma_3 + \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}) + \hbar\lambda(\sigma_+\hat{a} + \hat{a}^\dagger\sigma_-), \quad (9)$$

where the atomic operators are $\sigma_3 = |e\rangle\langle e| - |g\rangle\langle g|$; $\sigma_+ = |e\rangle\langle g|$; $\sigma_- = |g\rangle\langle e|$, the field (bosonic) operators obey $[\hat{a}, \hat{a}^\dagger] = 1$, and λ is the atom-field coupling constant. Here we are going to consider only the resonant case, i.e., the atomic transition frequency ω_0 equal to the field frequency ω. In the case of having the atom prepared in the excited state $|e\rangle$, and uncorrelated with the field at $t = 0$, the density operator (of the system atom-field) at a time t in the atomic basis is given by[14]:

$$\hat{\rho}(t) = \begin{pmatrix} \hat{A}\hat{\rho}_f(0)\hat{A}^\dagger & \hat{A}\hat{\rho}_f(0)\hat{B}^\dagger \\ \hat{B}\hat{\rho}_f(0)\hat{A}^\dagger & \hat{B}\hat{\rho}_f(0)\hat{B}^\dagger \end{pmatrix}$$

where $\hat{A} = \cos[\lambda t(\hat{a}\hat{a}^\dagger)^{1/2}]$, and $\hat{B} = -i\hat{a}^\dagger \sin[\lambda t(\hat{a}\hat{a}^\dagger)^{1/2}]/(\hat{a}\hat{a}^\dagger)^{1/2}$.

The density operator in equation (10) completely characterizes the system atom-field, and we can use it directly to evaluate some important expectation values. We can calculate, for instance, the probability of having the atom excited minus the probability of having it in the ground state, i.e., the so-called atomic inversion $W(t)$.

III.2 Atomic inversion

The atomic inversion can be written as $W(t) = \mathrm{Tr}[\hat{\rho}\,\sigma_3]$, and after a straightforward calculation using (10) we obtain:

$$W(t) = \sum_{n=0}^{\infty} P_n\{\cos[2\lambda t(n+1)^{1/2}]\}.$$

Despite the simplicity of the expression for $W(t)$, in general we have to employ numerical methods to obtain information from it. We clearly notice the dependence of the inversion on the photon number distribution of the initial field, P_n. The simplest case (not requiring numerical evaluation) is the one when the field is initially prepared in a number state $|m\rangle$, i.e., $P_n = \delta_{mn}$. Equation (11), then, yields simple sinusoidal oscillations, which means that the atom is periodically excited and de-excited, performing the well-known Rabi oscillations. The situation is more complicated if the field is prepared in a superpositon of number states, e.g, in a coherent state, whose P_n is given by:

$$P_n = \frac{\exp(-|\alpha|^2)|\alpha|^{2n}}{n!}. \tag{12}$$

which is a Poisson distribution for the n's. The dephasing and rephasing of the oscillations due to different photon numbers in equation (11) cause the Rabi oscillations to collapse and revive at characteristic times, which depend on the intensity of the field[15], and that we will call T_c and T_r, respectively. The collapse and revival phenomenon, which is basically due to the spread of quanta in a coherent state, also occur if the initial field is prepared, for instance, in a squeezed coherent state[11,12,9] and in superpositions of coherent states[5]. However, specific features of the photon distributions of those states of the field, such as oscillations, will cause a variety of atomic responses. If the field is initially prepared in a squeezed coherent state, its photon number distribution is given by:

$$P_n = \frac{\mathcal{N}}{n!\cosh r}\left(-\frac{1}{2}\tanh r\right)^n \exp[-\alpha^2(1+R\tanh r)]\mathcal{H}_n(A)^2,$$

for the particular case we are considering here, i.e., a real squeezing parameter r, and a real coherent amplitude α. The argument A is defined in equation (8). In this case, the atomic response will strongly depend on the sign of the squeezing parameter r, and ringing revivals will appear for $r > 0$ due to the characteristic oscillations in P_n [12,9]. It would be convenient now to define a scaled time $T_s = \lambda t/\left(2\pi\sqrt{\alpha^2+\sinh^2 r}\right)$, in such a way that we have a well defined revival at T_s. In Fig. 3a there is a plot of the atomic inversion as a function of the scaled time T_s, from numerical evaluation of (11), having the field initially in a squeezed coherent state with $\alpha = 5$ and $r = 1.5$. We clearly see the ringing revivals accompanying the main ones, as well as the longer collapse time for the Rabi oscillations. In Fig. 3b, we have the same atomic inversion, but with $r = -1.5$. We notice a quite different response, as it was already pointed out[11]. The broader photon distribution when $r < 0$ is responsible for the irregular atomic response (revivals not well defined), which actually resembles the response as if the initial field was prepared in a thermal state[16]. If the field is prepared in a superposition of squeezed coherent states, the revival time is reduced by one half, which corresponds to the appearence of additional revivals, as in superpositions of coherent states[5]. This happens because of the absence of odd photon numbers in the initial field, the time needed for a rephasing of the various Rabi oscillations involved will be approximately half of the time needed for a Poissonian distribution[5]. In Fig. 3c ($r > 0$), and 3d ($r < 0$), we have plots of the atomic inversion versus the scaled time T_s, using the photon number distribution in equation (7) and numerically evaluating (11). We would like to point out that despite the irregularity of the Rabi oscillations when $r < 0$, additional revivals, due to the quantum interference in the superposition state, can still be noticed.

We can identify, then, features arising from the "coherent part" of the superposition, such as halving of the revival time, and features due to the squeezing of the constituent states, such as ringing revivals, simultaneously present in the atomic response in the case considered here. Nevertheless, these effects seem to occur independently, i.e., there is no apparent interference between them.

IV. Evolution of the cavity field

IV.1 Field entropy

We are now interested in examining some aspects of the cavity field. Atom and field can be treated as separate subsystems, and their corresponding reduced density operators are obtained by the following tracing operation:

$$\hat{\rho}_{f(a)}(t) = \mathrm{Tr}_{a(f)}[\hat{\rho}(t)],$$

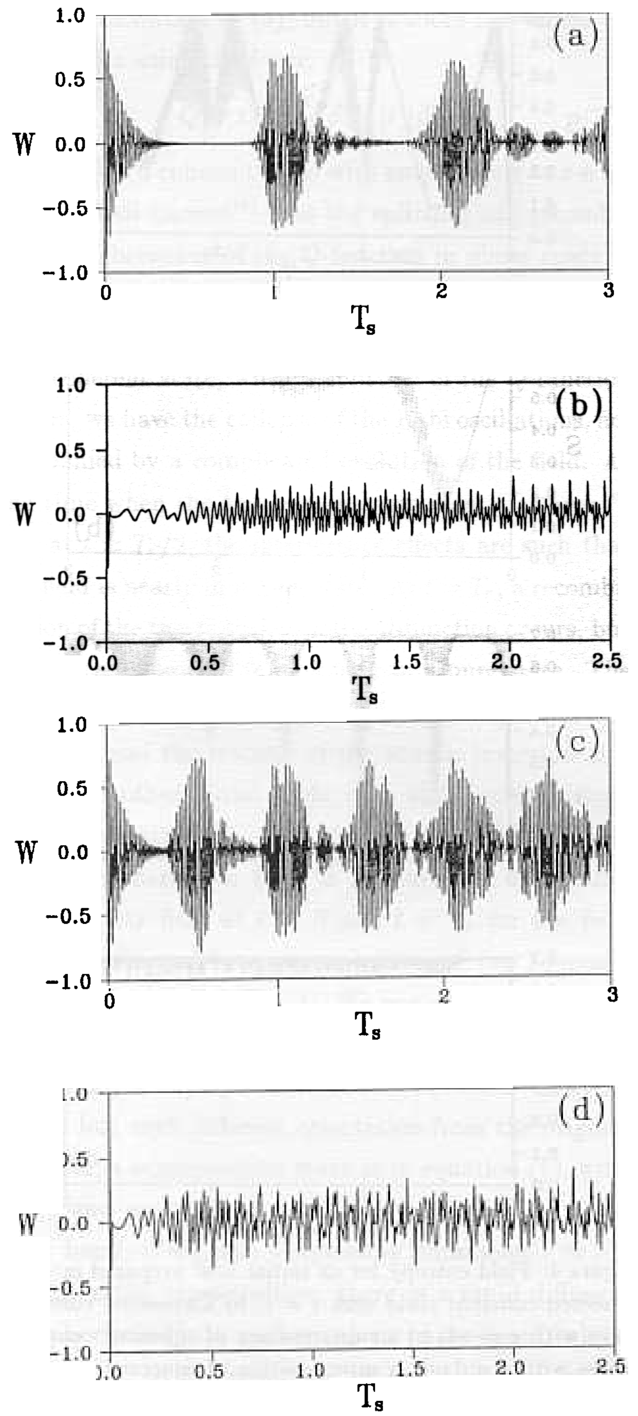

Figura 3: Atomic inversion for an initial field prepared in: a) a squeezed coherent state with $r = 1.5$; b) a squeezed coherent state with $r = -1.5$; c) a superposition of squeezed coherent states with $r = 1.5$; d) a superposition of squeezed coherent states with $r = -1.5$. In all cases $\alpha = 5$.

performed on the total density operator $\hat{\rho}(t)$ in equation (10). We are assuming that the field is not correlated to the atom at $t = 0$, and that each one is in a pure state. However, as soon as the interaction starts, the resulting entangled state is such that the state represented by (14) is no longer a pure state, at least during most of the time. A way of measuring the degree of purity of the field is through the so-called von Neumann entropy, defined, for each one of the subsystems, as[17]:

$$S_{f(a)} = -\mathrm{Tr}\left\{\hat{\rho}_{f(a)}(t)\ln[\hat{\rho}_{f(a)}(t)]\right\} \qquad (15)$$

The entropy is zero for any quantum mechanical pure state, assuming non-zero positive values for mixed states, and reaching a (possible) maximum value that depends on the number of available states in the system. In our case the maximum possible value for the entropy is $S_{max} = \ln 2 \approx 0.69$. According to the Araki-Lieb theorem[18], provided that the total initial entropy is zero (closed system), the entropy of the subsystems will be equal, i.e., $S_f = S_a$. We are going take advantage of this fact, because in the present case it is more convenient to calculate the atomic entropy rather than the field entropy. From the total density operator in (10), we obtain:

$$\hat{\rho}_a(t) = \mathrm{Tr}_f[\hat{\rho}(t)] = \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{12}^* & \lambda_{22} \end{pmatrix}$$

where

$$\lambda_{ij} = \sum_{n=0}^{\infty}\langle n|\rho_{ij}|n\rangle. \qquad (17)$$

Having diagonalized the atomic density operator, the von Neumann entropy is simply given by:

$$S_a = S_f = -\sum_{j=+,-}\eta_j\ln(\eta_j),$$

where

$$\eta_{\pm} = \frac{1}{2}\left\{1 \pm \left[(\lambda_{11}-\lambda_{22})^2 + 4|\lambda_{12}|^2\right]^{1/2}\right\}$$

It is well-known[14] that if the field is initially prepared in a coherent state, it almost returns to a pure state approximately at half of the revival time ($T_r/2$). It has also been found[9] that if the field is prepared in a squeezed coherent state, instead, the field is almost pure at both half of the revival time and at the revival time itself. This can be seen in Fig. 4a, where we have a plot of the field entropy (numerically evaluated) as a function of the scaled time T_s for an initial squeezed coherent state with $r = 1$ and $\alpha = 5$. In Fig. 4b, the situation is the same, but with $r = -1$, instead. We immediately notice a decrease in the degree of field purity. In this case it is more difficult for the evolution to "reconstruct" a pure state, and in our opinion, this happens because the field is super-Poissonian ($r < 0$) instead of sub-Poissonian ($r > 0$). If we now have the initial field prepared in the superposition state $|\Psi\rangle$ defined in equation (1), i.e., $\hat{\rho}(0) = |\Psi\rangle\langle\Psi|$, the situation is very different. For instance, for a superposition state having $\alpha = 5$ and $r = 1$ in both constituent states, the field entropy presents strong oscillations, that reach minimum values approximately at the corresponding revival times. This feature, already found when the field is prepared in a superposition of coherent states[5], can be appreciated in Fig. 4c. However, there is an important difference between the present example and the coherent state case studied in Reference [5]. The maximum degree of field purity is greater in the former than in the latter, and this also can be understood using phase-space representations of the field. We are using the squeezing parameter $r = 1$ precisely because, as it has been already pointed out[9], it corresponds to an "optimum value" (when $\alpha = 5$), for which the field almost returns to its initial features (a squeezed coherent state) approximately at the revival time. For these particular values of r and α, the field is extremely sub-Poissonian, so that both atom and field can almost return to their initial configurations at certain times. The revivals of the atomic inversion, for instance, are more "complete", in the sense that the atom is almost able to reach its initial level population as the initial field approaches an ideal sub-Poissonian state[9]. We notice that, at least regarding the field entropy, this is also true if the field starts in the superposition state (1). On the other hand, this reconstruction is not possible if the orientations of the constituent states are changed, in such a way the initial field is super-Poissonian ($r = -1$). In this case, as we see in Fig. 4d, the field is rapidly reduced to a mixed state, remaining in this condition as time goes on.

We would also like to note that the non-diagonal matrix elements in (16) are zero if the field is initially prepared in a superposition of squeezed states, unlike when it is prepared in a single squeezed state. For that reason the maximum purity reached by the field during the evolution will be always smaller in the former case than in the latter, fact that can be seen by comparing Fig. 4c with 4a.

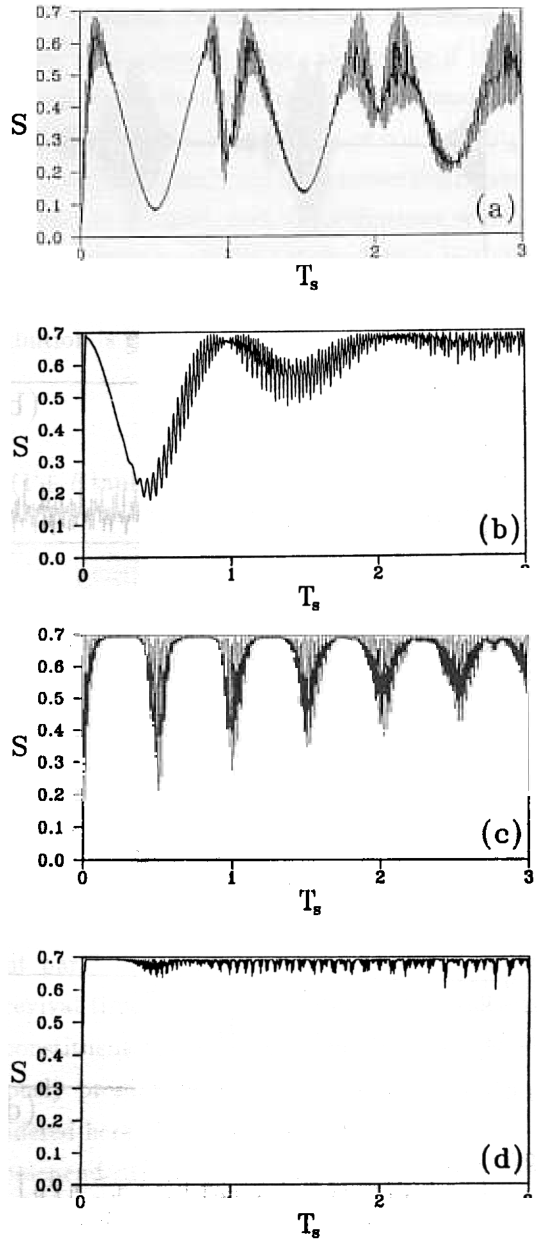

Figura 4: Field entropy for an initial field prepared in: a) a squeezed coherent state with $r = 1$; b) a squeezed coherent state with $r = -1$; c) a superposition of squeezed coherent states with $r = 1$; d) a superposition of squeezed coherent states with $r = -1$. In all cases $\alpha = 5$.

IV.2 Phase-space representation

A very clear way of depicting the field evolution in the JCM is through the phase-space representations (or quasiprobability distributions)[13]. The Q-function is especially suitable for that, and in fact it has been extensively used in this kind of problems[5,9,19]. The Q-function of a field described by $\hat{\rho}_f$ can be obtained from its definition in (3), but it is more convenient to calculate it using the form:

$$Q(\beta;t) = \frac{1}{\pi}\langle\beta|\hat{\rho}_f(t)|\beta\rangle, \qquad (20)$$

where $|\beta\rangle$ is a coherent state with amplitude $\beta = x + iy$.

It is well-known[19] that the splitting and recombination of branches of the Q-function in phase space is associated with the collapses and revivals of the atomic inversion. For instance, if the field is initially prepared in a coherent state, when a splitting of the Q-function occurs, we have the collapse of the Rabi oscillations, accompanied by a complicated evolution of the field. At the time when the two branches are most "far apart", i.e., at $t = T_r/2$, the interference effects are such that the field is nearly in a pure state. At $t = T_r$, a recombination of the two branches in the Q-function occurs, but in this case the field is no longer in a pure state. The correspondence between the collisions of Q-function's branches and the revivals of the atomic inversion also occurs for other initial fields, e.g., with superpositions of coherent states[5]. This general behavior will also be present here. In Fig. 5 we have the Q-function of the cavity field at $t = 0$ and $t = T_r$ for the field initially prepared in a superposition of two squeezed coherent states with $r = 1$. We notice that in fact at $t = T_r$ the field has evolved towards a configuration resembling a superposition of two squeezed coherent states, but with different orientation from the original ones, i.e., a superposition state as in equation (1), with $r < 0$ and an imaginary amplitude $\alpha' = i|\alpha|$. On the other hand, if we take a squeezing parameter $r = -1$ for the initial superposition, there is a rapid diffusion along a circle (with radius=α) in phase-space, which corresponds to the reduction of the field to a mixed state. We would like to remark that there is a clear connection between the orientation, the fluctuations in photon number of the (initial) constituent states and the way they are reduced to statistical mixtures. It is easier for the field to almost recover its purity during the evolution if $r > 0$ (sub-Poissonian), rather than if $r < 0$ (super-Poissonian) in the initial superposition state, and this corresponds to a splitting of the distribution along the direction in which there is already an increase in quadrature noise. For times long enough, however, there is an even spread of the field Q-function along a ring-like structure in both cases, as the field becomes basically a statistical mixture.

V. Conclusions

We have presented an investigation of a new class of quantum superposition states of the quantized electromagnetic field, that is, superpositions of squeezed coherent states. We have mainly discussed the interaction of such non-classical states of the field with matter, using a simple model of optical resonance (the Jaynes-Cummings model). The constituent states themselves (squeezed states), already show remarkable statistical properties, like oscillations in the photon number distribution, but because of the quantum interference, some non-classical properties are even more pronounced in a superpostition of two squeezed coherent states. Due to the inherent phase-sensitivity of the squeezed states, we are allowed to construct an infinite number of different types of superpositions, each one leading to a different evolution of the system atom-field. We have concentrated our analysis upon two particular superposition states, both having the corresponding constituent states oriented either along the x (real) or the y (imaginary) axes in phase-space, and we have noticed that there are two distinct types of oscillations in the photon number distribution of the superposition states considered here. We were also able to verify the consequences of having such non-classical features in the initial cavity field by simple inspection of the atom (field) evolution. The collapses and revivals of the atomic inversion, for instance, are very clearly affected by the different types of oscillations in the photon number distributions present in the superposition state. We would also like to recall an interesting feature concerning the evolution of the cavity field. We found that it is possible to obtain a more effective return of the field to an almost pure state if we have a superposition of squeezed coherent states with an "optimum" squeezing parameter $r \approx 1$ (if $\alpha = 5$), for which its photon number distribution is the narrowest possible (most sub-Poissonian). The existence of such an squeezing parameter for the initial

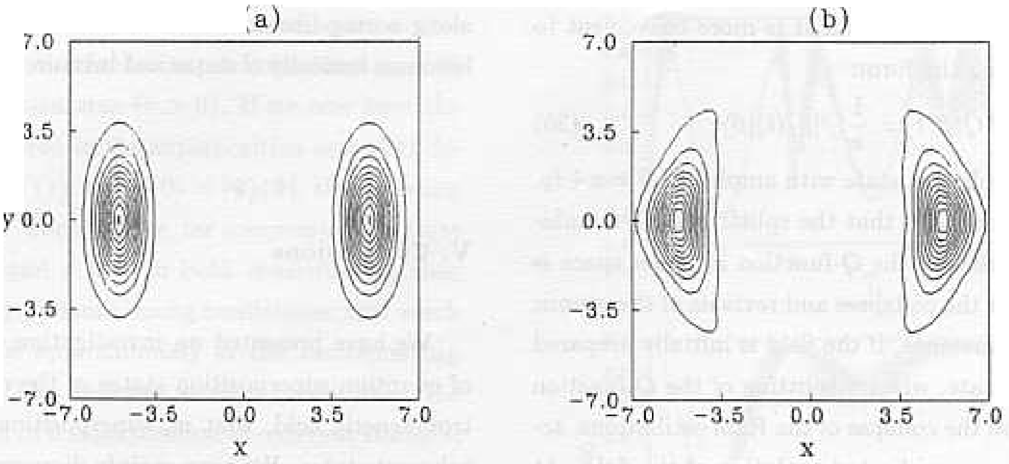

Figura 5: Q-function contours for an initial field prepared in a superposition of squeezed coherent states with $r = 1$ and $\alpha = 5$, at: a) $t = 0$; b) $t = T_r$.

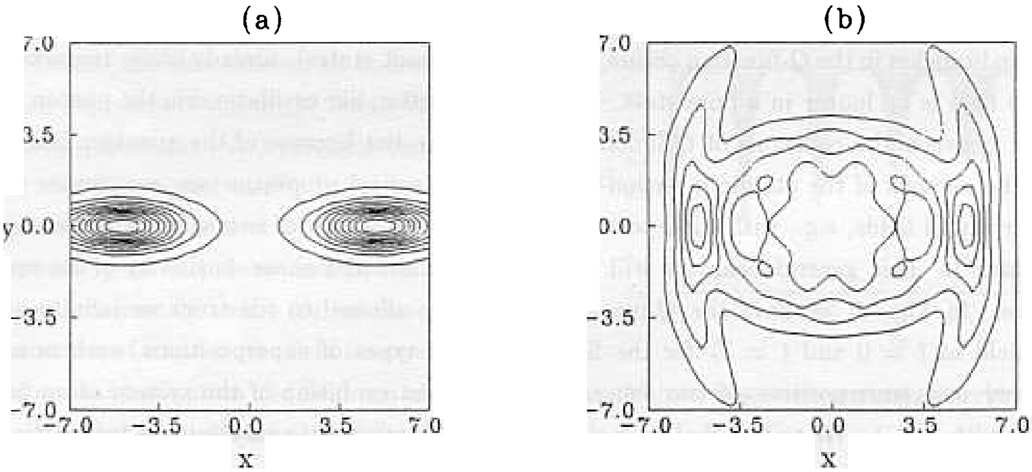

Figura 6: Q-function contours for an initial field prepared in a superposition of squeezed coherent states with $r = -1$ and $\alpha = 5$, at: a) $t = 0$; b)t= T_r.

state of the field related to its purity during the evolution has been already devised in a work concerning only a single squeezed coherent state as an initial field in the JCM[9]. From the phase-space point of view, this is related to the fact that there is less amount of noise in the quadrature "perpendicular" to the direction in which the bifurcation in phase-space occurs, that is, the $\hat{X}$ quadrature in our case. Otherwise there is just a dramatic reduction of the state of the field towards a statistical mixture.

Acknowledgments

This work was supported by CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico, Brazil), and by CONACyT (Consejo Nacional de Ciencia y Tecnología, Mexico).

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