Non-Markovian particle dynamics in continuously controlled quantum gases

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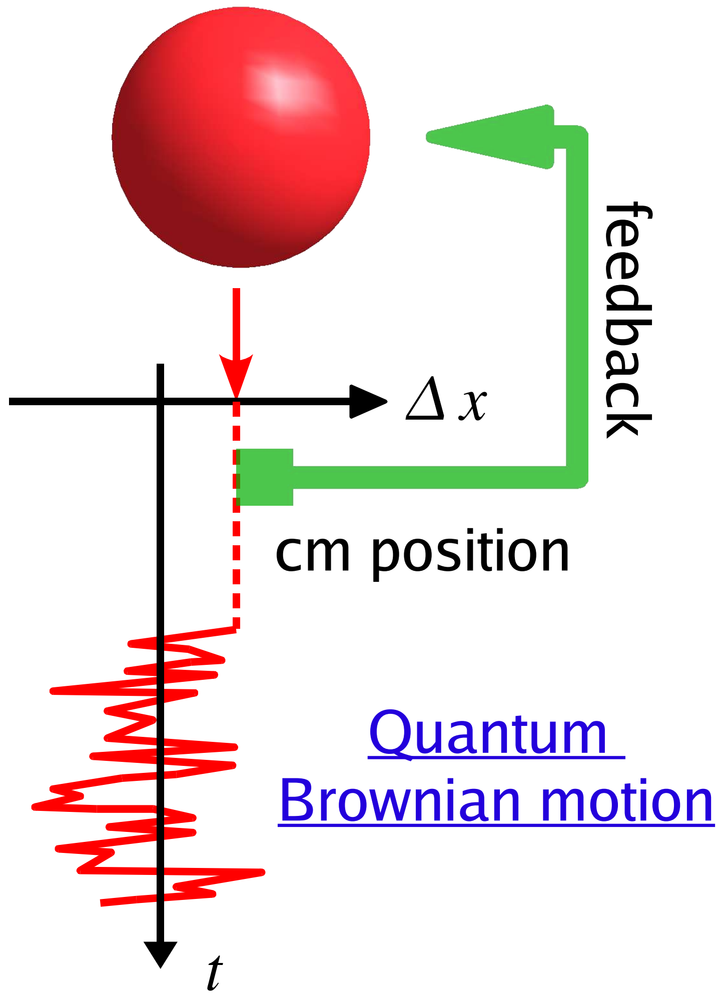
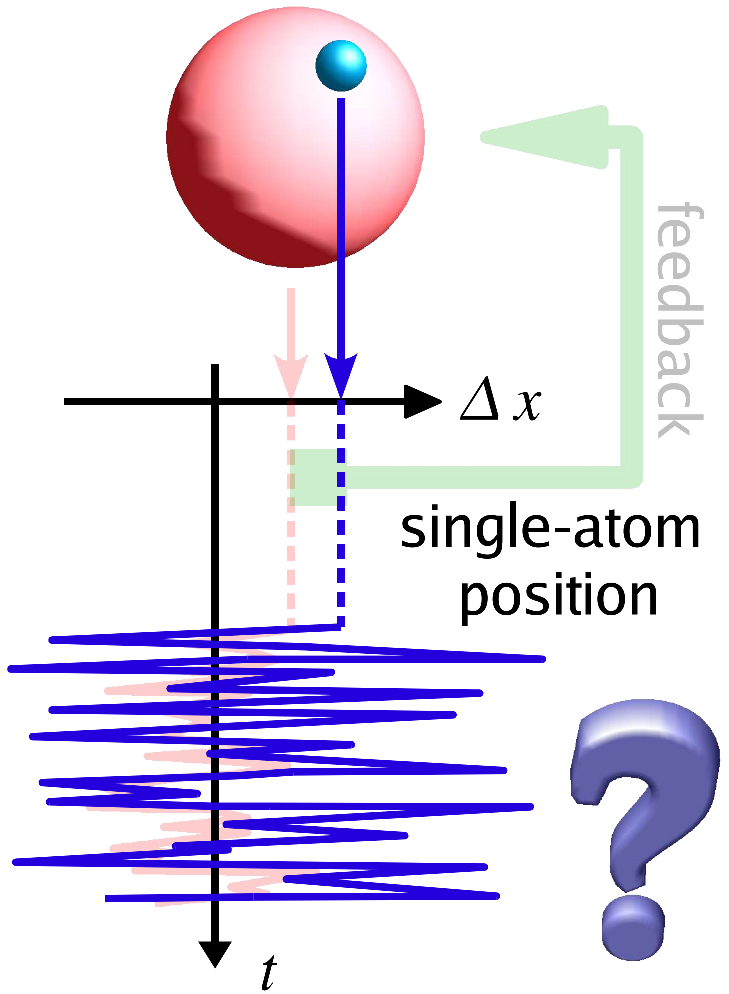
Motivation:





Localising object in lab = feedback
[Caves & Milburn, PRA **36**, 5543 (1987)]

Motion of single atom in the object?

Macroscopic view – Continuous feedback

Feedback loop:

- Measure centre of mass with finite resolution,
- Shift back centre of mass by a fraction.

Continuous limit: Rate of feedback $\gamma \to \infty$

- Master equation for many-atom density operator (Brownian motion) [Barchielli, Nuovo Cimento B **74**, 113 (1983)]:

$$\begin{aligned}\dot{\hat{\varrho}} &= -\frac{i}{\hbar}[\hat{H},\hat{\varrho}] + i\frac{\zeta}{2\hbar}[\hat{P},\{\hat{X},\hat{\varrho}\}] \\ &\quad -\frac{1}{8\sigma^2}[\hat{X},[\hat{X},\hat{\varrho}]] - \frac{\zeta^2\sigma^2}{2\hbar^2}[\hat{P},[\hat{P},\hat{\varrho}]].\end{aligned}$$

$\sigma^2 = \sigma_0^2/\gamma$ = time-integrated measurement variance.

$\zeta = \zeta_0 \gamma$ = feedback-shift rate.

Macroscopic view – Dynamics

Behaviour of centre of mass and total momentum can be easily obtained from the Master equation:

- No need for considering N-atom problem.
- Standard calculation for a single system with total mass $M = N\,m$.

In case of harmonically trapped atoms (stationary solution):

- Average centre-of-mass and total momentum vanish
- Variances become:

$$(\Delta X)^2 \rightarrow (\Delta X_0)^2 (\eta + \eta^{-1})/2$$
$$(\Delta P)^2 \rightarrow (\Delta P_0)^2 \{\eta[1 + (\zeta/\omega_0)^2] + \eta^{-1}\}/2$$

$\eta = (\Delta X_0)^2 / (\zeta\,\sigma^2)$ = trap localisation / feedback localisation.

ω_0 = trap frequency.

Problem – Many-atom correlations

Quantum measurement of collective centre of mass:

- Creates (quantum) correlations between atoms.
- Single-atom description only with approximations (typical many-particle problem).

True interactions between atoms also create correlations!

- Consider an ideal gas for simplicity,
- Pure effects due to feedback.

It is shown here, that: [Ivanov & Wallentowitz, Phys. Rev. Lett. (in press)]

- Single-atom motion is analytically solvable via trick,
- Motion is fed by noise,
- Noise is coloured due to atomic correlations.
- ***Single-atom motion is non-Markovian!***

Microscopic view – Single-atom density matrix

Indistinguishable atoms

- Use second-quantized bosonic matter field:

$$[\hat{\phi}(x), \hat{\phi}^\dagger(x')] = \delta(x - x')$$

Relation between macroscopic and microscopic description:

$$\hat{X} = \frac{1}{\langle \hat{N} \rangle} \int dx\, \hat{\phi}^\dagger(x)\, x\, \hat{\phi}(x)$$

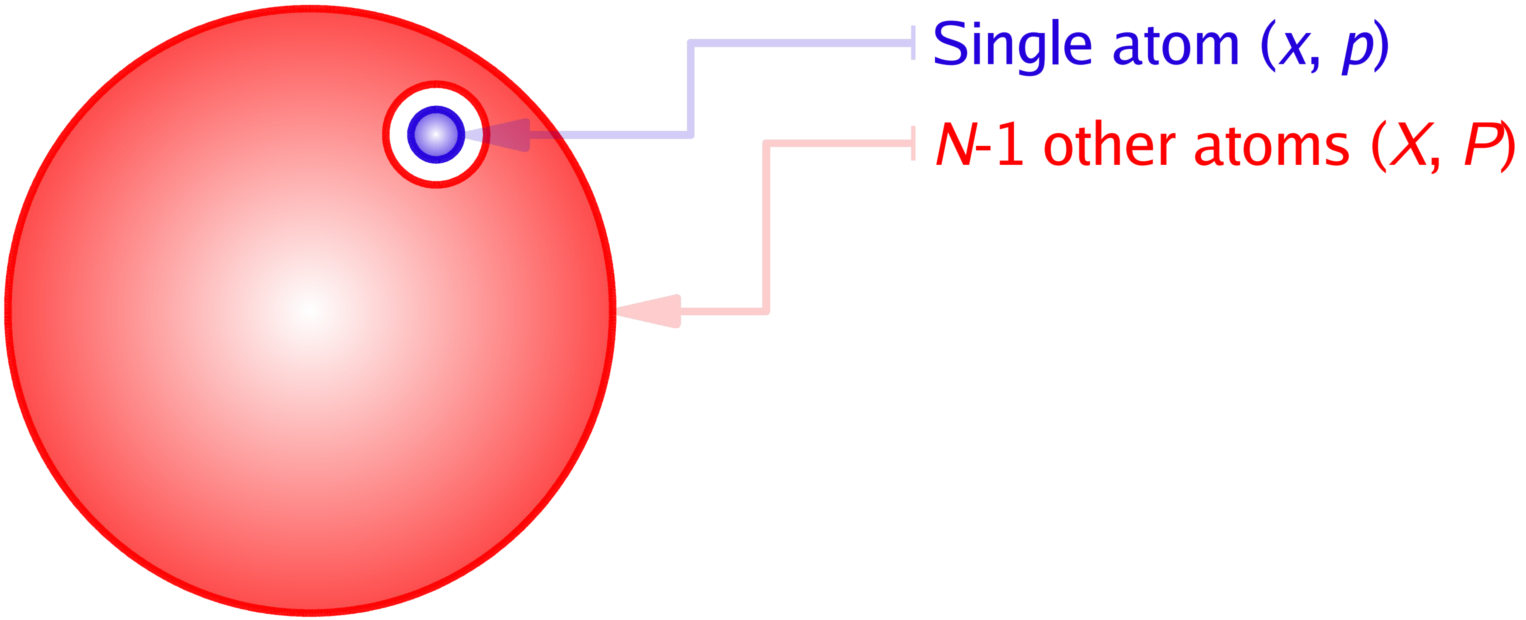
$$\hat{P} = -i\hbar \int dx\, \hat{\phi}^\dagger(x)\, \frac{\partial}{\partial x}\, \hat{\phi}(x)$$

Function of interest: ***reduced single-atom density matrix***

$$\rho(x, x') = \langle \hat{\phi}^\dagger(x')\, \hat{\phi}(x) \rangle$$

Two-body description – Classical idea

Consider single atom in its surrounding (classical picture):



Quantum picture: + Symmetrisation of N-atom wavefunction

Two-body description – Wigner function

Wigner function for single atom + centre of mass of N-1 "other" atoms:

$$W_N(x,p;X,P) = (2\pi\hbar)^{-4}\int dx' \int dX' \int dP' \int dS$$
$$\times\ \exp\{-i\left[x'p+PX'+XP'+(N-1)S\right]/\hbar\}$$
$$\times \left\langle \underbrace{\hat{\phi}^{\dagger}\!\left(x-\frac{x'}{2}\right)}_{\text{single atom}} \underbrace{e^{i(\hat{P}X'+\hat{X}P'+\hat{N}S)/\hbar}}_{\text{quasi centre of mass}} \underbrace{\hat{\phi}\!\left(x+\frac{x'}{2}\right)}_{\text{single atom}} \right\rangle .$$

Properties of the Wigner function:

- Trace over (X, P, N) gives single-atom Wigner function,
- Trace over (x, p) gives Wigner function only of quasi centre of mass.

- Single-atom dynamics is okay, but subtlety with centre of mass!
- (X, P) = formal extension of single-atom Hilbert space!

Analytic solution

Two-body Wigner function: ***Why and for what purpose?***

Answer:

- Wigner function obeys a closed Fokker-Planck equation.
- Is of linear type with positive semi-definite diffusion matrix.

▶ Gaussian Green's function & ***bound analytic solutions!***

Instead of Fokker-Planck equation use equivalent “Langevin” equations.

Advantages:

- Identify coupling between microscopic and macroscopic systems.
- Properties of the driving noise sources.

Stochastic differential equations

Stochastic trajectory conditioned on having *N* atoms in the system:

$$
\begin{aligned}
dx_N &= \left[\frac{p_N}{m} - \zeta\left(\overbrace{\frac{x_N}{\langle\hat{N}\rangle}}^{\text{small}} + X_N\right)\right]dt + \zeta\sigma\, d\xi_1,\\
dp_N &= -m\omega_0^2 x_N\, dt + \frac{\hbar}{2\sigma\langle\hat{N}\rangle}\, d\xi_2,\\
dX_N &= \left[\frac{P_N}{M} - \zeta\Theta_N\left(\overbrace{\frac{x_N}{\langle\hat{N}\rangle}}^{\text{small}} + X_N\right)\right]dt + \Theta_N\zeta\sigma\, d\xi_1,\\
dP_N &= -M\omega_0^2 X_N\, dt + \frac{\hbar\Theta_N}{2\sigma}\, d\xi_2,
\end{aligned}
$$

$M = m\langle\hat{N}\rangle$ = total mass $\qquad$ $\Theta_N = (N-1)/\langle\hat{N}\rangle$ = correction factor

ξ_1, ξ_2 = statistically independent Wiener processes

Elimination of macroscopic variables

Decoupling in the limit of large atom number <*N*>:

Solve for *X*(t), *P*(t) and insert into *dx(t) = ... + ... X(t) dt + ...* .

- Deterministic driving force.
- Addition to the noise source!

Effective noise source for *dx*: = Wiener + Ornstein-Uhlenbeck process

$$d\xi_N(t) = d\xi_1(t) - 2\Gamma_N dt \int_0^t \Big\{ d\xi_1(t')\cos[\Omega_N(t-t')] + \frac{\eta\, d\xi_2(t') - \alpha_N\, d\xi_1(t')}{\sqrt{1-\alpha_N^2}} \sin[\Omega_N(t-t')] \Big\} e^{-\Gamma_N(t-t')}$$

Parameters: $\Gamma_N = \zeta\Theta_N/2$, $\Omega_N^2 = \omega_0^2 - \Gamma_N^2$, $\alpha_N = \Gamma_N/\omega_0$

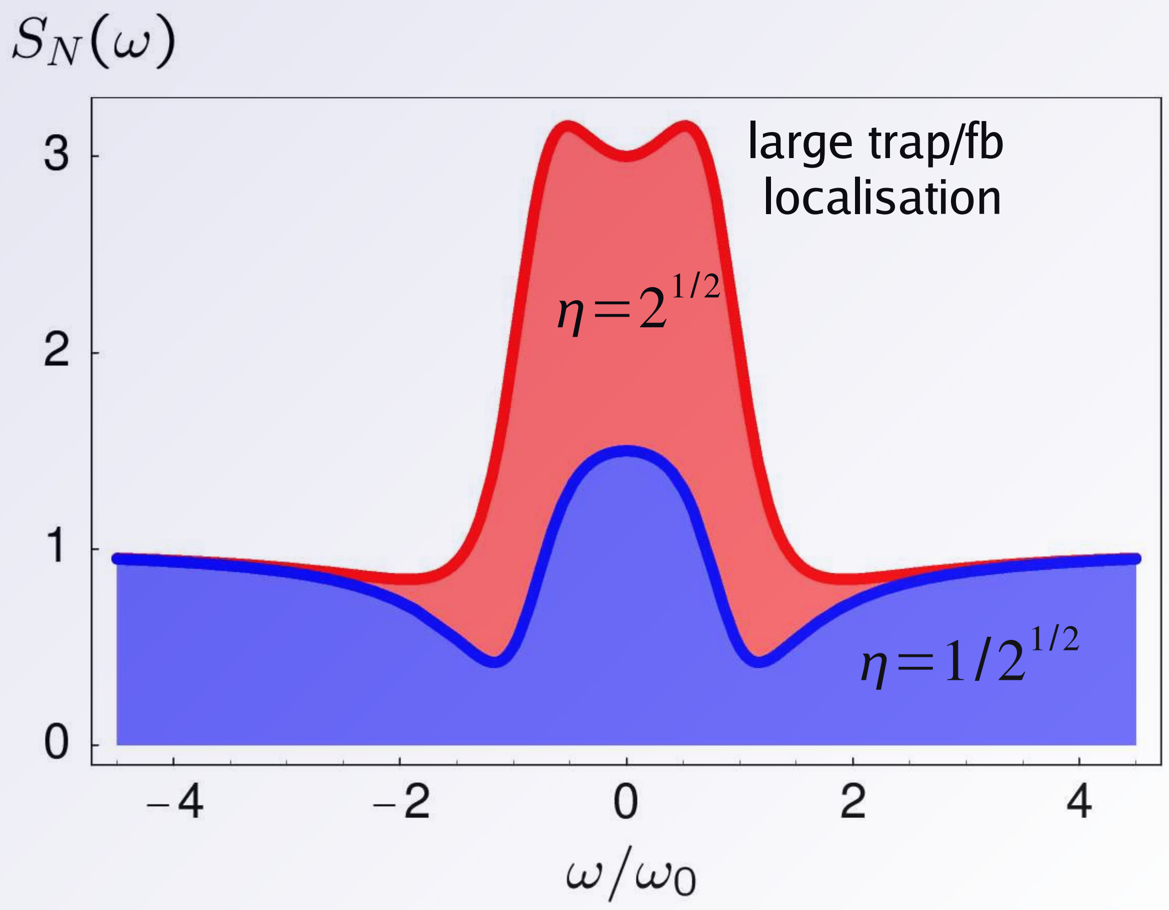
- ***Single-atom dynamics is fed by coloured noise !***

Coloured noise spectrum

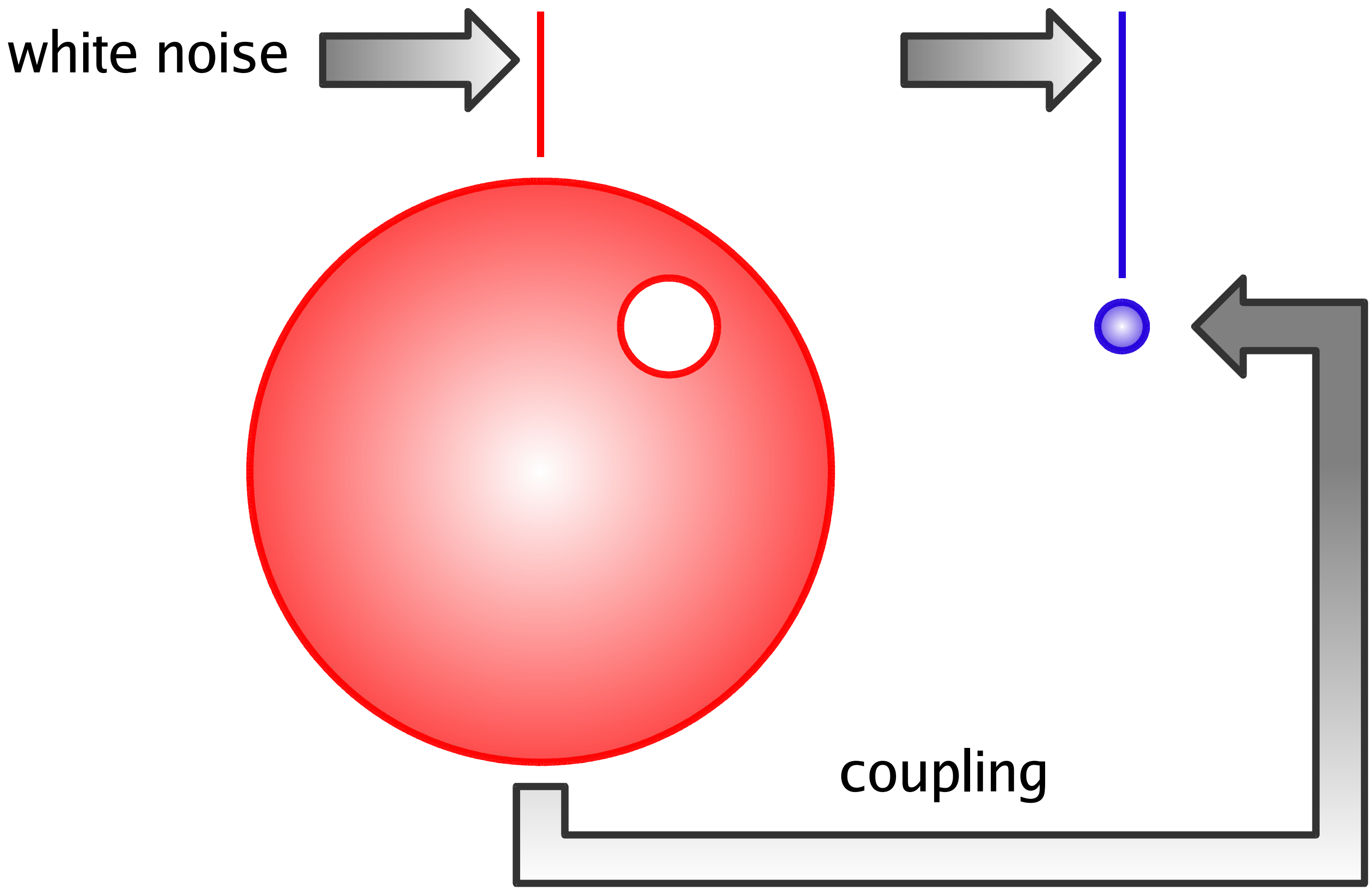
Fourier transform of stationary correlation function:

$$S_N(\omega) = \lim_{t\to\infty} \int d\tau\, e^{i\omega\tau}\, \overline{\dot{\xi}_N(t+\tau)\dot{\xi}_N(t)}$$

Small feedback-induced damping: $\alpha_N < 1/2^{1/2}$



Explanation of noise reductions



Driven, damped harmonic oscillator:

Destructive phase shift near resonance frequency!

Measurable effects – Density profile

Asymptotic behaviour for large times: density oscillations

$$\Delta x^2(t) \to \Delta X^2 + C(t)$$

Oscillating correlation function:

$$\begin{aligned} C(t) &= \left\langle \int \frac{dx}{N}\, \hat{\phi}^\dagger(x)\, [q(x, \partial_x, t)]^2\, \hat{\phi}(x) \right\rangle \\ &- \left\langle \int \frac{dx}{N}\, \hat{\phi}^\dagger(x) q(x, \partial_x, t) \hat{\phi}(x) \int \frac{dx'}{N}\, \hat{\phi}^\dagger(x') q(x', \partial_{x'}, t) \hat{\phi}(x') \right\rangle \end{aligned}$$

Freely oscillating quadrature:

$$q(x, \partial_x, t) = x \cos(\omega_0 t) - \frac{i\hbar \partial_x}{m\omega_0} \sin(\omega_0 t)$$

Cauchy-Schwarz inequality: ***Quantumness if***

- $C(t) < 1$ ***Observable as density oscillation below*** ΔX !

Summary & conclusions:

What has been shown:

- Analytic solution of a correlated many-atom problem
- Two-body approach
- Single atom is fed by coloured noise
- Non-Markovian trajectories in phase space
- Observable via density oscillations

Many-atom correlations have been cast into colour of noise feeding the single atom!